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NUMERICAL EXPERIMENTS ON THE ONSET OF LAYERED CONVECTION IN A NARROW SLOT CONTAINING A STABLY STRATIFIED FLUID

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Abstract—Numerical experiments on the initial formation, growth, and merging of convective layers in a density stratified solute solution contained in a narrow slot with steadily applied lateral temperature gradient are reported on. At supercritical conditions, the kinetic energy of the layers, once formed, steadily increases until viscous interaction leads to the eventual merging of adjacent layers.

The stability of the initial layer formation is studied. The critical thermal Rayleigh number for neutral conditions is determined through numerical experiment in close agreement with previous linear analysis and experimental results.

NOMENCLATURE

D_S, solute diffusivity;
D_T, thermal diffusivity;
g, gravitational accelerating;
h, layer height;
H, integration region height;
J, Jacobian;
KE, kinetic energy;
L, slotwidth;
Le, fluid Lewis number;
n, number of layers observed;
Pr, fluid Prandtl number;
Ra, thermal Rayleigh number;
Rs, solute Rayleigh number;
S, solute concentration;
t, time;
T, temperature;
u, horizontal velocity;
v, vertical velocity;
x, horizontal coordinate;
y, vertical coordinate.

Greek symbols
α, coefficient of expansion due to T;
β, coefficient of expansion due to S;
ΔT, wall temperature increase;
ν, kinetic viscosity;
ρ, fluid density;
ϕ_0, initial solute stratification;
ψ, stream function;
ω, vorticity.

1. INTRODUCTION

WE ARE interested in investigating the resulting convection process which occurs when a linearly stratified salt solution contained in a tall slot of width L, is exposed to a carefully applied lateral temperature gradient. It has been previously found that under certain conditions, characterized by thermal and solute Rayleigh numbers, that the quiescent fluid departs from the case of lateral and vertical transport of heat and salt due only to conduction and diffusion respectively and forms a convecting, layered system. This process is part of a general class of two-component diffusion phenomena which have been recently reviewed by Turner [1]. In our investigation, heat/salt and heat/sugar are the diffusing components, having a ratio of diffusivities of $Le = D_T/D_S \doteqdot 100$ and 281 respectively. In this investigation, we are interested in ascertaining whether finite difference techniques may be successfully used in investigating the onset and time dependent growth of the initial layered system, and in determination of the limits, in terms of a thermal Rayleigh number, of the initial layered system.

Steplike structures in temperature and salinity have been observed in the Northeast Atlantic [2], the Arctic Ocean [3], Antartic Lakes [4], the Indian Ocean [5], near Bermuda [6], and at the bottom of the Red Sea [7]. Since layered systems produce significant departures in the vertical transport of heat and salt, an understanding of these processes is crucial in studying the local and global air–sea heat balance. Turner ([8] Chapter 8) has discussed the oceanographical implications of layered systems. Furthermore, it has recently been shown experimentally that lateral heating, along with other lateral driving mechanisms, lead to situations in double diffusive phenomena where both diffusive and salt finger type instabilities coexist [9].

The formation of a layered flow of a stably stratified fluid contained in a narrow slot with steadily applied lateral temperature gradient has been previously studied by Blumsack [10], Thorpe *et al.* [11], and Hart [12, 13] using various linearized stability approaches. Experiments on the formation, evolution, and breakdown of these layered systems have also been reported by the above using heat/salt and heat/sugar systems.

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Similar layered systems form when a stably stratified solution is exposed to a suddenly applied lateral temperature gradient. This time dependent problem has been previously studied experimentally [14] and using numerical simulation [15, 16].

In the following section we write down the differential equations and boundary conditions to be solved where the fluid motion is assumed to be two-dimensional, based on previous investigations of the problem [10–14]. Our approach has been to deal with the non-linear form of the governing equations using explicit finite difference techniques. Our solution algorithm, which employs a conservative difference form of the stream function, vorticity, and energy equations, is outlined in Section 3. Convection in an initially quiescent fluid is instigated through the input of a disturbance in the fluid vorticity field. This approach has been used by others on other fluid stability problems. For example, Samuels and Churchill [17] studied the onset of convection in a two-dimensional enclosure heated from below. More recently, Chen [16] used a similar approach to investigate the stability of the wide gap problem with suddenly applied lateral temperature gradient as mentioned above.

In Section 4 we compare the results of our numerical simulation with an experiment reported by Hart [13] using a heat/sugar system. Also, the stability limits of heat/salt systems calculated by the finite difference approach are compared with the predictions of linear theory [11, 12] and laboratory experiments [11]. Finally, in Section 5 we discuss the advantages and limitations of this numerical approach.

2. GOVERNING EQUATIONS

A schematic diagram of a section of the slot is shown in Fig. 1. The region has slot width L and height H where the expected fluid motion is assumed periodic over the distance H. The region aspect ratio (H/L) is selected so that H is some integer multiple of the expected layer height, h. The side walls are impermeable to S. The left vertical wall is isothermal at temperature T_0; the right wall is maintained at $T_0+\Delta T$. The solution contained between the walls is assumed incompressible with solute concentration S such that its state equation is given by

$$\rho = \rho_0[1-\alpha(T-T_0)+\beta(S-S_0)] \quad (1)$$

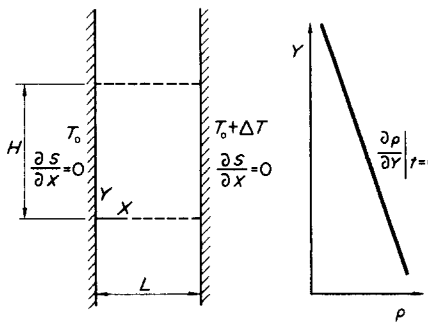

FIG. 1. Schematic diagram of a section of a tall narrow slot containing a linearly stratified solution.

where

$$\alpha = -\frac{1}{\rho_0}\frac{\partial \rho}{\partial T}\bigg|_{S_0} \qquad \beta = \frac{1}{\rho_0}\frac{\partial \rho}{\partial S}\bigg|_{T_0} \quad (2)$$

are the coefficients of expansion for heat and solute. The subscript 0 denotes a reference state.

If the fluid motion is restricted to two dimensions in the x, y plane with horizontal and vertical velocities u, v, and if the usual Boussinesque approximation is applied while compressibility effects and viscous dissipation are neglected, then the governing equations for laminar flow are:

Vorticity Transport:

$$\frac{\partial \omega}{\partial t} = J_{xy}(\omega,\psi)+\nu\nabla^2\omega - g\left[\alpha\frac{\partial T}{\partial x}-\beta\frac{\partial S}{\partial x}\right] \quad (3)$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} = J_{xy}(T,\psi)+D_T\nabla^2 T \quad (4)$$

Conservation of Solute:

$$\frac{\partial S}{\partial t} = J_{xy}(S,\psi)+D_S\nabla^2 S \quad (5)$$

Relation between Stream Function and Vorticity:

$$\nabla^2\psi = -\omega \quad (6)$$

where ω is the two-dimensional fluid vorticity defined by

$$\omega = \frac{\partial u}{\partial y}-\frac{\partial v}{\partial x} \quad (7)$$

and the stream function, ψ, has been introduced such that,

$$u = -\frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x}. \quad (8)$$

ν, D_T, D_S are the kinematic viscosity, thermal diffusivity, and solute diffusivity respectively. J_{xy} is the Jacobian representation of the convection terms.

With the boundary conditions on T shown in Fig. 1, and in the absence of fluid motion, the steady state temperature of the fluid due to conduction only is

$$T(x) = T_0+\frac{\Delta T}{L}x. \quad (9)$$

If the temperature of the fluid departs from the pure conduction case (that is, when layers form), it can be represented as

$$T(x,y,t) = T(x)+T_p \quad (10)$$

where T_p is the temperature perturbation. Similarly, if the slot initially contains a linearly stratified salt solution with specified initial stable gradient,

$$\phi_0 = \frac{\partial S}{\partial y}\bigg|_{t=0} \quad (11)$$

then upon formation of a layered system, the salt concentration can be represented as

$$S = S_0+\frac{\alpha\Delta T}{\beta L}x+\phi_0(y-H)+S_p \quad (12)$$

where S_p is a perturbation quantity. Equation (12) is obtained through the requirement that $\partial\rho/\partial x = 0$ in the absence of layers.

If we substitute equations (10) and (12) into equations (3)–(5), we arrive at the following set of equations in terms of flow and perturbation quantities

$$\frac{\partial\bar{\omega}}{\partial\bar{t}} = J_{\bar{x}\bar{y}}(\bar{\omega},\bar{\psi}) + Pr\nabla^2\bar{\omega} - RaPr\left[\frac{\partial\bar{T}_p}{\partial\bar{x}} - \frac{\partial\bar{S}_p}{\partial\bar{x}}\right] \quad (13)$$

$$\frac{\partial\bar{T}_p}{\partial\bar{t}} = J_{\bar{x}\bar{y}}(\bar{T}_p,\bar{\psi}) + \nabla^2\bar{T}_p + \frac{\partial\bar{\psi}}{\partial\bar{y}} \quad (14)$$

$$\frac{\partial\bar{S}_p}{\partial\bar{t}} = J_{\bar{x}\bar{y}}(\bar{S}_p,\bar{\psi}) + \frac{1}{Le}\nabla^2\bar{S}_p + \frac{\partial\bar{\psi}}{\partial\bar{y}} - \frac{Rs}{LeRa}\frac{\partial\bar{\psi}}{\partial\bar{x}} \quad (15)$$

where we have introduced dimensionless variables as follows:

$$\bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{L}, \quad \bar{t} = \frac{tD_T}{L^2}$$
$$\bar{T}_p = T_p/\Delta T, \quad \bar{S}_p = \frac{\beta S_p}{\alpha\Delta T} \quad (16)$$
$$\bar{u} = \frac{uL}{D_T}, \quad \bar{v} = \frac{vL}{D_T}, \quad \text{etc.}$$

In equations (13)–(15), $Pr = \nu/D_T$ is the fluid Prandtl number, $Le = D_T/D_S$ is the Lewis number, $Ra = g\alpha\Delta TL^3/\nu D_T$ and $Rs = g\beta\phi_0 L^4/\nu D_S$ are the thermal and solute Rayleigh numbers where $Rs < 0$ is a stable solute stratification since $\phi_0 < 0$ in equation (12) for solute concentration decreasing upward. In the following, we drop the overbar notation; all variables are dimensionless unless stated otherwise.

Just prior to layer formation, the fluid in the slot is assumed to be quiescent with steady state conduction T and S profiles such that $\partial\rho/\partial x = 0$ at any level. Therefore, for initial conditions, we use

$$\left.\begin{aligned} \omega(w,y,0) = \psi(x,y,0) = 0 \\ T_p(x,y,0) = S_p(x,y,0) = 0 \end{aligned}\right\} \quad (17)$$

Side boundary conditions consistent with Fig. 1 are:

$$T_p({}^0_1,y,t) = \frac{\partial S_p}{\partial x}({}^0_1,y,t) = 0 \quad (18)$$

together with the no-slip condition. (We arbitrarily set $\psi({}^0_1,y,t) = 0$ along the no-slip side boundaries.) Finally, we assume that the solution is periodic over height $\bar{H}$ giving

$$\left.\begin{aligned} S_p(x,0,t) &= S_p(x,H,t) \\ T_p(x,0,t) &= T_p(x,H,t) \\ \omega(x,0,t) &= \omega(x,H,t) \\ \psi(x,0,t) &= \psi(x,H,t) \end{aligned}\right. \quad (19)$$

3. THE NUMERICAL SIMULATION

The specific details of our finite difference scheme are described in detail elsewhere [18]. The simulation of equations (13)–(15) used explicit forward time differencing. It is considered to be first order accurate in time and second order accurate in spatial coordinates. The convective terms, J_{xy} were represented by the second order accurate and conservative equations developed by Arakawa [19]. The Laplacian terms were represented by the usual five point operator. The source terms, $\partial T_p/\partial x$, $\partial S_p/\partial x$, $\partial\psi/\partial x$, etc., were approximated using centered differences about each point. The upper and lower periodic boundary conditions were incorporated into the solution through suitable modification of the finite difference representation of equations (13)–(15) written along the region boundary. Since all quantities on the right-hand sides of equations (13)–(15) were written at the previous time level, each equation at each grid point had only one unknown; thus, a straightforward marching technique of solution was employed as follows:

1. Initiate calculation with small random field of vorticity.
2. Solve for corresponding stream function (equation 6). This is accomplished using Hockney's Fourier Analysis and Cyclic Reduction algorithm [20].
3. Calculate new internal grid point values of ω (equation 13), S (equation 14), and T (equation 15).
4. Calculate side boundary values of ω using extrapolation formulas suggested by Aziz and Hellums [21].
5. Calculate flow kinetic energy, etc. Increment time and return to step 2 or stop.

Further comment on step one is necessary. In any mathematical treatment of a flow instability we assume that infinitesimal perturbations of all wave number exist initially and that the disturbance flow with the appropriate wave number will grow most rapidly and predominate the flow. Numerically, the same approach may be used with two exceptions. First, the initial disturbance is no longer infinitesimal, but rather it is numerically small compared to the final flow field. For our layered convection flows, the initial disturbance kinetic energy is 5 to 10 orders of magnitude smaller than the final flow kinetic energy at supercritical conditions.

The second difference from the analytical approach is that the structure (i.e. wave number) of the initial disturbance must *a priori* be assumed in the numerical approach. Elder [22] has used trigonometric functions for the initial disturbance in a numerical study of roll convection in a stratified fluid heated from below. Samuels and Churchill [17] used a localized disturbance. A less restrictive form for the initial disturbance is to use a spatially random field because it, on the average, tends to cancel itself out and not force the developing flow field to have any particular predetermined characteristics. Chen's [16] investigation of the stability of a stratified solute in a wide gap exposed to impulsive changes in wall temperature showed that the use of initial random fields of ω, T, or S are all equivalent in producing the layered convection. In our investigation, we have arbitrarily chosen to initiate the disturbance flow with a random field of vorticity.

Finally, we have monitored the growth or decay of the evolving flow by calculating the dimensionless kinetic energy per unit depth as

$$KE = \int_0^H \int_0^1 \left[\left(\frac{\partial \psi}{\partial x} \right)^2 + \left(\frac{\partial \psi}{\partial y} \right)^2 \right] dx\,dy. \quad (20)$$

Equation (20) was evaluated using the trapezoid rule applied to an area integral.

4. RESULTS

Preliminary calculations were performed for the case of homogeneous (single component) free convection between two infinite isothermal vertical plates at different temperatures. The total steady state kinetic energy predicted from the numerical simulation using 9×9 and 9×17 grids was found to be within 1·6 per cent of the theoretically calculated value. Most subsequent calculations reported here are for 9×17 grids.

We have performed numerical calculations for heat/salt ($Le = 101$) and heat/sugar ($Le = 281$) systems. Our numerical simulation, corresponding to supercritical heat/sugar layer formation reported by Hart [13], will be discussed below.

Hart's apparatus consisted of a 1 cm wide × 40 cm high slot which for the case considered contained a linearly stratified sugar solution such that $-Rs = 2{\cdot}6 \times 10^7$. He increased the side wall temperature difference very slowly (over a period of 4–5 h) up to the point where cell formation was evident, as indicated through a dye flow visualization technique. The wall temperature difference, in the case considered $\Delta T = 8{\cdot}5°C$, was then held constant. For the purposes of our numerical simulation this corresponds to an initial supercritical thermal Rayleigh number, Ra, of $1{\cdot}59 \times 10^5$. We set the region height at $H = 1{\cdot}57$, which is three times the roll height predicted using the relations developed in [13] for incipient roll formation at $(-Rs) = 2{\cdot}6 \times 10^7$.

Figure 2 shows stream function contour maps, traced from computer generated contour maps, for four selected times during initial layer formation, growth, and merging. Initially, three sets of opposite rotating layer systems form, consistent with the prediction of linear theory. Each system has a dimensionless layer height of approximately 0·52. The upper set is weaker than the others probably due to our selection of the wrong value of H for this set of supercritical solute and thermal Rayleigh numbers. By $t = 0{\cdot}161$ counterclockwise rotating layers dominate the opposite rotating ones. The strength of fluid rotation has also increased, as evidenced by increases in maximum stream function. This is, perhaps, more clearly depicted in Fig. 3, where the vertical velocity profile across the central height cell is plotted. In Fig. 3, we observe over an order of magnitude increase in maximum fluid velocity between $t = 0{\cdot}161$ and $t = 0{\cdot}202$. Using

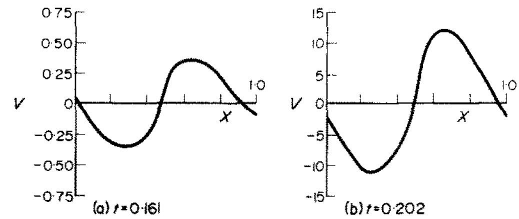

FIG. 3. Dimensionless vertical velocity profiles for a layer at supercritical conditions as listed in Fig. 2.

$L = 1$ cm, $D_T = 1{\cdot}5 \times 10^{-3}$ cm^2/s, we estimate the vertical velocities in the layer to range between 0 to 0·0005 cm/s at $t = 0{\cdot}161$ (1·8 min) and from 0 to 0·02 cm/s at $t = 0{\cdot}202$ (2·2 min). These numerically generated estimates bracket Hart's estimate, based on visual observations, of velocities of the order of 0·001 cm/s during the initial stages of cell formation. By $t = 0{\cdot}202$ the original six rotating layers have been merged into 3 counterclockwise rotating ones due to the continuous input of energy in excess of that required to maintain the original system. This compares favorably with the physical observation of initial layer merging at a nondimensional time of between 0·18 and 0·27 as read from Hart's Fig. 7.

The numerical simulation may be used to assess the limits of stability in terms of thermal and solute Rayleigh numbers for layer formation. This can be used as a comparison with previously advanced linear calculations. Figure 4 shows the dimensionless kinetic energy for supercritical, neutral, and subcritical thermal Rayleigh number for a heat/salt system with $(-Rs) = 5 \times 10^6$, and $\bar{H} = 1$. In each case, the initial starting perturbation on the vorticity field produced a dimensionless kinetic energy of 10^{-5}. This initial random perturbation rapidly decays with rapid oscillations. For the subcritical condition ($Ra = 10^4$), there is insufficient input of thermal energy to overcome the potential energy of the stable stratification; the solution rapidly becomes quiescent. For supercritical Rayleigh number ($Ra = 5 \times 10^4$) the random field rapidly gives way to coherent layer formation at $t \cong 0{\cdot}06$. Since there is excess energy input, the kinetic energy of the newly formed layers steadily increases, leading to eventual over-driving of the layers and merging, as discussed above. At critical conditions, there is just

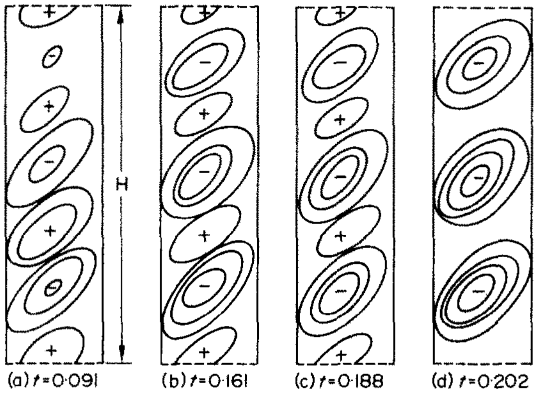

FIG. 2. Stream function contour maps with $H = 1{\cdot}57$, $(-Rs) = 2{\cdot}6 \times 10^7$, $Ra = 1{\cdot}59 \times 10^5$, $Le = 281$, $Pr = 6{\cdot}7$. In each case six-isovalues are sketched between the following limits: (a) $-1{\cdot}0 \times 10^{-3} < \psi < 1{\cdot}08 \times 10^{-3}$, (b) $-1{\cdot}31 \times 10^{-1} < \psi < 1{\cdot}28 \times 10^{-1}$, (c) $-1{\cdot}18 < \psi < 6{\cdot}71 \times 10^{-1}$, (d) $-4{\cdot}27 < \psi < 9{\cdot}21 \times 10^{-1}$.

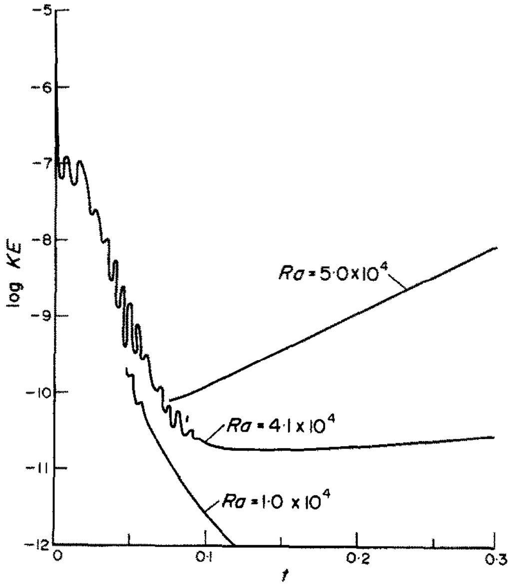

FIG. 4. Dimensionless flow kinetic energy for subcritical, neutral, and supercritical conditions with $(-Rs) = 5 \times 10^6$, $H = 1$, $Le = 101$.

sufficient thermal energy input to maintain the layers against the stable stratification. Vertical convection is balanced by opposing diffusion of solute; the kinetic energy remains constant once the layers have formed.

In actual practice, it is difficult to select Ra for exactly zero kinetic energy growth. Therefore, we arbitrarily define the neutral condition, for a given height H, as that Ra which produces

$$\left|\frac{dKE}{dt}\right| \leqslant 10^{-2}. \tag{21}$$

For a given solute Rayleigh number $(-Rs)$, we must also iterate on the region height in order to determine the critical thermal Rayleigh number which would form in a tall slot. For fixed initial solute stratification,

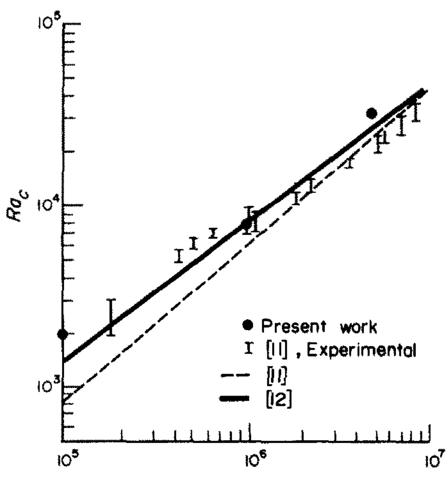

FIG. 5. Neutral stability curve for layer formation in heat/salt systems ($Le = 101$) compared with linear theories and previous experiments.

and hence $(-Rs)$, that value of H which produces the smallest neutral Ra is judged to be the critical thermal Rayleigh number for layer formation. We have performed this calculation for three solute Rayleigh numbers for a heat/salt system. The results are compared with the linear stability calculations of Thorpe *et al.* [11], and Hart [12], along with THS's experimental results in Fig. 5. Our predictions compare favorably with the experimental observations up to $(-Rs) = 10^6$ and appear high at $(-Rs) = 5 \times 10^6$.

Ideally, we should be able to predict the layer height from our calculations. The dimensionless layer height should be given by

$$h = \frac{H_c}{n} \tag{22}$$

where n is the number of layer pairs initially produced by the simulation and H_c is the region height at neutral Ra. At $(-Rs) = 10^5$ this calculation produces layer pair heights in agreement with observation. However, at higher $(-Rs)$ the smaller layer heights which should be produced according to previous observations apparently cannot be resolved by only seventeen grid points in the vertical. The finite difference simulation produces an aliased result where the kinetic energy goes into larger wave length cells. This is probably the reason for the larger than expected critical Ra obtained at $(-Rs) = 5 \times 10^6$.

5. CONCLUSIONS

Based on our numerical experiments, we have shown that the essential features of convective layer formation and growth in a laterally heated stratified fluid may be simulated using simple finite difference techniques. Our simulation indicates that the kinetic energy of layers formed at supercritical conditions continuously increases until viscous interaction leads to the merging of adjacent layers. This picture is in general agreement with Hart's [13] observations based on his experiments.

We have also investigated the use of finite difference techniques in the study of the onset of flow instability. We have found that a neutral stability curve may be constructed through numerical simulation of the full, nonlinear set of governing equations. However, the problem of disturbance wavelength selection, in particular at large solute Rayleigh numbers, remains. This problem would not occur in a vertical slot of finite height since the presence of slot top and bottom would determine the number of rolls formed at supercritical conditions. However, in attempting to simulate a very tall slot, or one of infinite height, the selection of a height, H, over which the flow is assumed to be periodic is necessary; dictated by reasons of economy of computer storage. This height, H, must be varied by trial and error until one which is an exact integer multiple of the roll height occurring for fixed Ra, and Rs, is obtained. Once the "correct" height, H, is obtained, its value may be checked by rerunning the calculation at $2H$. If n rolls of height h were produced using region height H, then $2n$ rolls of height h must be produced when using region height $2H$.

At large solute Rayleigh numbers the rolls produced have small height, h, and the behavior of the numerical solution becomes very sensitive to small variations in H. Due to the use of a finite grid, aliasing errors tend to shift the energy which would normally go into the growth of short wave components of the solution into a long wavelength solution.

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REFERENCES

1. J. S. Turner, Double-diffusive phenomena, *Ann. Rev. Fluid Mech.* **6**, 37–56 (1974).
2. R. I. Tait and M. R. Howe, Some observations of thermohaline stratification in the deep ocean, *Deep-Sea Res.* **15**, 275–280 (1968).
3. V. T. Neal, S. Neshyba and W. Denner, Thermal stratification in the Arctic Ocean, *Science* **166**, 373–374 (1969).
4. R. A. Hoare, Problems of heat transfer in Lake Vanada, a density stratified Antarctic lake, *Nature, Lond.* **210**, 787–789 (1966).
5. H. Stommel and K. N. Fedorov, Small scale structure in temperature and salinity near Timor and Mundanao, *Tellus* **19**, 306–325 (1967).
6. J. W. Cooper and H. Stommel, Regularly spaced steps in the main thermocline near Bermuda, *J. Geophys. Res.* **73**, 5849–5854 (1968).
7. E. T. Degens and D. A. Ross, The Red Sea hot brines, *Scient. Am.* **222**(4), 32–53 (1970).
8. J. S. Turner, *Buoyancy Effects in Fluids.* Cambridge University Press, Cambridge (1973).
9. J. S. Turner and C. F. Chen, Two-dimensional effects in double-diffusive convection, *J. Fluid Mech.* **63**, 577–592 (1974).
10. S. L. Blumsack, Formation of layers in a stably stratified fluid, Geophys. Fluid. Dyn. Summer Notes II, W.H.O.I. (1967).
11. J. A. Thorpe, P. K. Hutt and R. Soulsby, The effect of horizontal gradients on thermohaline convection, *J. Fluid Mech.* **38**, 375–400 (1969).
12. J. E. Hart, On sideways diffusive instability, *J. Fluid Mech.* **49**, 279–288 (1971).
13. J. E. Hart, Finite amplitude sideways diffusive convection, *J. Fluid Mech.* **59**, 47–64 (1973).
14. C. F. Chen, D. G. Briggs and R. A. Wirtz, Stability of thermal convection in a salinity gradient due to lateral heating, *Int. J. Heat Mass Transfer* **14**, 57–65 (1971).
15. R. A. Wirtz, D. G. Briggs and C. F. Chen, Physical and numerical experiments on layered convection in a density stratified fluid, *Geophys. Fluid Dynam.* **3**, 265–288 (1972).
16. C. F. Chen, Onset of cellular convection in a salinity gradient due to a lateral temperature gradient, *J. Fluid Mech.* **63**, 563–576 (1974).
17. M. R. Samuels and S. W. Churchill, Stability of a fluid in a rectangular region heated from below, *A.I.Ch.E. Jl* **13**, 77–85 (1967).
18. L. H. Liu, Numerical simulation of convective layer formation in a slot, MIE Report No. 004, MIE Department, Clarkson College of Technology, Potsdam, New York (1974).
19. A. Arakawa, Computational design of long-term numerical integration of the equations of fluid motion: two-dimensional incompressible flow. Part I, *J. Comput. Phys.* **1**, 119–143 (1966).
20. R. W. Hockney, The potential calculation and some applications, *Meth. Computat. Phys.* **9**, 135–212 (1970).
21. K. Aziz and J. D. Hellums, Numerical solution of the three dimensional equations of motion for laminar natural convection, *Physics Fluids* **10**, 315–324 (1967).
22. J. W. Elder, Numerical experiments with thermohaline convection, *Physics Fluids* **12**, Supp. II, 194–197 (1969).

EXPERIENCES NUMERIQUES SUR L'APPARITION DE LA CONVECTION EN COUCHE DANS UNE CAVITE ETROITE CONTENANT UN FLUIDE STRATIFIE STABLE

Résumé—On présente des expériences numériques sur la formation initiale le développement et la dispersion de couches convectives dans une solution stratifiée en densité contenue dans une cavité étroite sur laquelle est appliqué un gradient latéral de température constant. Dans des conditions supercritiques, l'énergie cinétique des couches une fois formées, croit uniformément jusqu'à ce que l'interaction visqueuse conduise à la fusion éventuelle de couches adjacentes.

La stabilité de la formation de la couche initiale est étudiée. Le nombre de Rayleigh thermique critique pour les conditions neutres déterminé par des expériences numériques est en bon accord avec les résultats antérieurs d'analyse linéaire et expérimentaux.

NUMERISCHE EXPERIMENTE ÜBER DEN ANFANG VON "GESCHICHTETER" KONVEKTION IN EINER SCHMALEN NUT, DIE EIN STABIL GESCHICHTETES FLUID ENTHÄLT

Zusammenfassung—Es wird von numerischen Experimenten berichtet über Anfangsformation, Anwachsen und Verschmelzen von konvektiven Schichten in einer je nach Dichte des gelösten Stoffes geschichteten Lösung, die in einem engen Spalt mit stetig aufgebrachtem seitlichen Temperaturgradienten enthalten ist. Bei überkritischen Bedingungen wächst die kinetische Energie der einmal gebildeten Schichten stetig an, bis viskose Wechselwirkungen schließlich zum Verschmelzen der benachbarten Schichten führen.

Die Stabilität der anfänglichen Schichtenformation wurde untersucht. Die kritische thermische Rayleigh-Zahl für neutrale Bedingungen wurde durch numerische Experimente in guter Übereinstimmung mit vorhergehender linearer Analyse und experimentellen Ergebnissen bestimmt.

ЧИСЛЕННЫЕ ЭКСПЕРИМЕНТЫ ПО ВОЗНИКНОВЕНИЮ СЛОИСТОЙ КОНВЕКЦИИ В УЗКОМ ЩЕЛЕВОМ КАНАЛЕ, СОДЕРЖАЩЕМ УСТОЙЧИВО СТРАТИФИЦИРОВАННУЮ ЖИДКОСТЬ

Аннотация — Описываются численные эксперименты по возникновению, росту и слиянию конвективных слоев в стратифицированном по плотности растворе, содержащемся в узком щелевом канале с постоянным боковым градиентом температуры. В закритической области кинетическая энергия образовавшихся слоев постоянно увеличивается до тех пор, пока вязкое взаимодействие не приведет к возможному слиянию соседних слоев.

Исследуется устойчивость начального образования слоев. Критическое тепловое число Релея для нейтральных условий определялось с помощью численных экспериментов в близком соответствии с ранее производимым линейным анализом и ранее полученными экспериментальными данными.